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Business Intelligence and Multi-market Competition ¹

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Abstract

We consider a multimarket framework where a set of firms compete on two oligopolistic markets. The cost of production of each firm allows for spillovers across markets, ensuring that output decisions for both markets have to be made jointly. Prior to competing in these markets, firms can establish links gathering business intelligence about other firms. These links have two effects. First the quality of the good produced by the firm which forms the link increases. Second the quality of the good of the firm which receives the link decreases. We characterize the business intelligence equilibrium networks and networks that maximize social welfare under the most interesting scenario of diseconomies of scope. We find that due to externalities, the equilibrium networks may be over-connected relative to socially optimal networks creating a role for policy intervention. We also find that in some situations firms may refrain from gathering information even if it is costless. Moreover, even though intelligence gathering leads to increased product quality, there exist situations where it is detrimental to both consumer welfare and social welfare.

JEL classification: C70; L13; L20.

Keywords: Oligopoly, Multimarket, Networks.

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Introduction

Firms routinely collect and make use of business information about their rivals. With increasing competition at the global level, modern firms keep tabs on each other by engaging in competitive intelligence gathering activities. Competitive intelligence is the name given to the systematic and ethical approach for gathering, analyzing and managing information that can affect a firm's plans, decisions and operations. In 2002 for instance, *Business Week* reported that 90% of large companies have competitive intelligence staff, and many large US firms spend more than \$1 million annually on competitive intelligence. Moreover, several major multinational firms like GM, Kodak and BP have their own separate competitive intelligence units.

Of course firms also spy on each through more nefarious means. For instance the American Society of Industrial Security (ASIS) released a survey stating that economic espionage grew by 323% between 1992 and 1996. In fact realizing the enormity of this problem, in 1996 the US Congress passed the *Economic Espionage Act*, and by 2005 the US Department of Justice was engaged in prosecuting 45 cases under this act. There are also instances where the distinction between legal and illegal intelligence gathering activities is blurred. Crane (2005, [5]) is an interesting study of three cases that virtually cross the realm of competitive intelligence to being illegal. Probably the most notorious case listed in this study is Procter and Gamble's attempt to find out more about Unilever's hair care business by hunting through their garbage bins. In fact numerous such tales about business spooks and their sordid activities can be found in the popular press demonstrating that firms attempt to access information about their competitors by hook or by crook.

Our reading of the literature in this area as well as the popular press suggests a number of stylized facts which we use in this paper. First, intelligence gathering whether legal or illegal is an issue of growing concern. Second, such activities are more likely in high-tech firms, the drug industry and the defense related sector. Typically it is also the case that such firms are involved in producing more than one product often with inter-related costs. This creates spillovers effects across markets which may be positive or negative. Third, firms are aware that their competitors are attempting to obtain information about them, and that their competitiveness will be nega-

tively affected by the information obtained. Finally, despite protective measures, rival firms are often able to engage in intelligence gathering successfully.

Our paper focuses on the pattern of intelligence gathering links between competitors in multimarket oligopolies and on the impact of these links on firms behavior. We model the spying behavior of firms as a two stage game and examine the interaction between the spillovers involved by spying activities and multimarket competition.

In the first stage, firms establish (directed) links with competitors in order to get information about the characteristics of the rivals' products. The information obtained by firm i through its link with firm j allows firm i to incorporate some valuable characteristics of firm j 's product in its own product. This has a positive impact on firm i 's competitiveness in the market. By contrast, firm's j competitiveness in the market is negatively affected by the link formed by i . One way to think about it is that due to its link firm i knows additional technical weaknesses of the good of firm j and will spread these negative information. Consequently the quality of the good of firm j , as perceived by the consumers, is lower. Link formation is costly capturing the fact that intelligence gathering is a costly activity.

In the second stage, firms play a Cournot game. Each firm in the model produces two different products with inter-related costs and is engaged in Cournot competition in two markets simultaneously. For much of the paper we focus on the more interesting case and assume that the cost function exhibits diseconomies of scope. Later in the paper we discuss the consequences of economies of scope.

To obtain insights about the role of intelligence gathering when there are negative spillovers or diseconomies of scope across markets, we begin by assuming that all firms engage in intelligence gathering in one market only. After solving for the Cournot equilibrium in the second stage, we look for the Nash equilibrium of the link formation, or intelligence gathering game. We show that only certain types of networks, namely the complete network, the empty network and the k -all-or-nothing-networks can be equilibrium networks.⁶ We also characterize the networks that

⁶The k -all-or-nothing-networks are networks where k firms have formed a link with all firms while the others have not formed any link.

maximize social welfare and show that equilibrium networks and efficient networks do not always coincide. The difference between the socially optimal and equilibrium networks is crucial since equilibrium networks do not take the externalities into account. Since different parameter ranges support different architectures as being socially optimal, the policy maker has to be aware of industry parameters before regulating the amount of corporate intelligence gathering activities in a particular industry. This observation is still valid when firms engage in intelligence gathering in several markets simultaneously.

We also provide a number of other interesting insights. We show that intelligence gathering activities do not always depend on the costs of these activities. This is because the spillover effect of production costs across markets can dominate the gains from new information. In some situations, firms will refrain from engaging in spying even if the costs of these activities are very low. Moreover, even though intelligence gathering leads to improvements in product quality, there exist situations where these activities are detrimental to consumers as well as social welfare. Lastly, it is interesting to observe that in some situations competitors may indeed wish to be spied upon. In other words in multimarket competition, we may expect to observe situations where firms do not try to prevent competitors from engaging in intelligence gathering directed at them.

Next, we extend our model to intelligence gathering activities in both markets. This leads to an increase in the number of possible equilibrium configurations without altering the above observations. Finally, we introduce economies of scope across markets. Since production costs provide positive spillovers across markets, costs of gathering intelligence determine how much of this activity takes place in equilibrium. Thus in this case the multimarket competition leads to the same qualitative outcome as a competition in a single market. In other words we show that business intelligence gathering is strongly influenced by the nature of spillovers across markets.

Our paper intersects two strands of existing literatures. It is related to the network formation models in oligopolistic settings found in the work of Goyal and Joshi (GJ, 2003, [6]), and Billand and Bravard (BB, 2004, [2]). GJ link formation requires mutual consent, and both firms involved in a link obtain resources from each other, while in the model of BB, link formation does need

consent, and only the firm establishing a link with another firm gets access to the resources from the latter. In other words, link formation as well as flows of resources are undirected in GJ, and directed in BB. Our model share some similarities with both models. First, in our model link formation is non cooperative, as in BB. Second, in our model the formation of a link has a direct impact on the two firms involved in the link, as in GJ. However, while in GJ the direct impact of a link on the two involved firms is positive, in our model, this link has a positive direct impact on one firm (the initiator of the link) and a negative direct impact on the other firm (receiver of the link). We believe that this is a realistic feature that captures well the nature of competition between firms. Note that in the GJ and BB models, firms compete only on one market and this difference in formulation alters the results significantly in our model. In particular in BB, the complete network is the unique equilibrium and efficient network when the cost of forming links is zero. By contrast, in our model, there are cases where even with zero link costs the empty network is the unique equilibrium network. Moreover, the complete network is not the only efficient network anymore.

Our paper is also related to the theory of multimarket competition, in particular to the work of Bulow, Geanakoplos, and Klemperer (1985, [4]) on multimarket oligopolies. These authors examine how a change in one market can have ramifications on a second market. In the Bulow et al. (1985, [4]) model changes are exogenous. By contrast, in our model while costs are inter-related, changes in quality are endogenous and depend on the choices firms make regarding their intelligence gathering activities. The paper also provides an interesting comparison with the traditional literature on multimarket competition where the focus is on mutual forbearance (see for instance Bernheim and Whinston, 1990, [1]). In our model with diseconomies of scope, we find that for certain parameters ranges firms may choose to spy on their rivals only on one market. This leads to a situation where every firm improves its quality and behaves aggressively on one market only allowing its competitors to do the same on the other market. This seemingly collusive behavior arises in equilibrium.

The rest of the paper is organized as follows. The model setup is presented in section 2. In section 3 we provide a characterization of equilibrium networks and section 4 analyzes the

efficient networks. Section 5 explores the implications of allowing firms to form links on both markets. In section 6 we discuss how the introduction of economies of scope across markets can affect the results and section 7 concludes.

1 The Model

In this section we introduce basic network concepts and describe the Cournot game played by the N firms in our setting.

1.1 Network Preliminaries

Let $N = \{1, \dots, n\}$, with $n \geq 3$, denote a set of *ex ante* identical firms. Each firm produces two products, and is simultaneously engaged in Cournot competition with all the other firms in both markets. We assume that each firm $i \in N$ can form links with the other firms before competing in both markets. For any $i, j \in N$, $g_{i,j} = 1$ implies that firm i has a directed link with firm j , while $g_{i,j} = 0$ denotes the absence of such a link. We denote the directed links vector of firm i by $\mathbf{g}_i = (g_{i,1}, \dots, g_{i,i-1}, 0, g_{i,i+1}, \dots, g_{i,n})$. We interpret the link from firm i to firm j as spying activity (or intelligence gathering) of i directed at j . A directed network $\mathbf{g} = \{(g_{i,j})_{i \in N, j \in N}\}$ is a formal description of the spying activities that exist between the firms. Let \mathcal{G} denote the set of all possible directed networks. Let $N_i(\mathbf{g}) = \{j \in N | g_{i,j} = 1\}$ be the set of firms j about whom i gathers information. Its cardinality is given by $n_i(\mathbf{g})$. Let $T_i(\mathbf{g}) = \{j \in N | g_{j,i} = 1\}$ be the set of firms j which spy on i . Its cardinality is given by $t_i(\mathbf{g})$. We denote by $n_{-i}(\mathbf{g}) = \sum_{j \neq i} n_j(\mathbf{g})$ the number of links in the network excluding the links originating from firm i , and by $t_{-i}(\mathbf{g}) = \sum_{j \neq i} t_j(\mathbf{g})$ the number of links in the network excluding the links pointing to firm i . Note that we have $\sum_{i=1}^n n_i(\mathbf{g}) = \sum_{i=1}^n t_i(\mathbf{g})$.

We now define the network architectures that are important for our analysis. A network is a k -all-or-nothing-network if k firms have formed links with all other firms, while the remaining $n - k$ firms have formed no links. The complete network and the empty network are special cases of this architecture. In the *complete* network for every pair of firms i and j there is a link

from i and j . A network \mathbf{g} is *empty* if no firm has formed links.

1.2 Links Formation and the Cournot Game

We consider two oligopoly markets labelled market 1 and market 2. Let q_i be the quantity produced by firm i on market 1 and Q_i be the quantity produced by firm i on market 2. Let $\mathbf{q} = (q_1, \dots, q_i, \dots, q_n)$ and $\mathbf{Q} = (Q_1, \dots, Q_i, \dots, Q_n)$ be the vectors of quantities produced by the n firms on market 1 and on market 2 respectively. Demand is assumed to be independent across markets.

We assume a representative consumer with the following quasi-linear aggregate utility function:⁷

$$U(\mathbf{q}, \mathbf{Q}, I) = u(\mathbf{q}) + v(\mathbf{Q}) + I, a \quad (1)$$

where,

$$u(\mathbf{q}) = \sum_{i=1}^n \alpha_i q_i - \frac{1}{2} \left(\sum_{i=1}^n q_i^2 + 2\delta \sum_{i=1}^n \sum_{j < i} q_i q_j \right),$$

and,

$$v(\mathbf{Q}) = \sum_{i=1}^n \beta_i Q_i - \frac{1}{2} \left(\sum_{i=1}^n Q_i^2 + 2\delta \sum_{i=1}^n \sum_{j < i} Q_i Q_j \right),$$

with $\delta > 0$ Consumers maximize utility on market 1 and on market 2, subject to the budget constraint $\sum_{i=1}^n p_i q_i + \sum_{i=1}^n P_i Q_i + I \leq R$, where R denotes income, p_i and P_i denote firm i 's products prices, on market 1 and on market 2 respectively.

Note that equation (1) is a version of the standard quadratic utility function introduced by Singh and Vives (1984, [9]), and discussed by Hackner (2000, [7]), when there are two independent markets. In this function, α_i and β_i represent the quality of the products sold by firm i on market 1 and market 2 respectively in a vertical sense. Other things equal, an increase in α_i increases the marginal utility of consuming good i . Parameter $\delta > 0$ could be interpreted in terms of

⁷The model structure is deliberately kept simple to keep the algebra tractable and also to obtain the precise spying architectures. More general formulations can be done that give the same qualitative results.

horizontal product differentiation. In this paper, we focus on the vertical product differentiation induced by intelligence gathering links, so we will suppose in the following that δ is close to 1. Finally, this utility function implies that consumers spend only a small part of their income on the two products ensuring that an interior solution exists.

In the two stage game played by the firms, stage 1 involves intelligence gathering through link formation and stage 2 is quantity competition. For the time being in stage 1 we assume that firms can form links only on the first market. A link represents gathering information about competitors' products and costs $f > 0$. This in turn allows the firm gathering the information to increase the quality of its product to be sold on market 1. As a result, demand for this firm on this market goes up. By contrast, some technical weaknesses of the firm being spied will be known and spread and the demand for this firm on market 1 goes down. We assume that *ex ante* firms are symmetric in market 1. Observe that in our context the consumer can get the same utility from the products of two different firms, even though the two products are not the same. This for instance could be true for cell phones where one might have a longer battery life while the other has better media capabilities, the two products giving the consumer the same level of satisfaction. Since the product have different attributes, spying on rivals can help firms improve the quality of their products. In our model, firm i 's product quality depends both of the number of firms with whom i has formed a link or spies on and the number of firms which has formed a spying link with i . More specifically, in the remainder of the paper, we assume the following specific form for the product quality function:⁸

$$\alpha_i = \gamma + \gamma_0 n_i(\mathbf{g}) - \gamma_1 t_i(\mathbf{g}),$$

We assume that $\gamma > \gamma_1(n-1)$. Further, as in BKG (1985, [4], pg. 490-491) in our model costs of firms are interrelated across markets in the following quadratic way:

$$CT(q_i, Q_i) = \frac{1}{2}(q_i + Q_i)^2.$$

⁸This is a natural adaptation of the marginal cost formulation used by Bloch (1995, [3]) or Goyal and Joshi (2003, [6]) to the quality production function, when the negative impact of being spied on is introduced.

Thus, the cost incurred by firm i depends on the quantities produced in both markets and there are joint diseconomies across markets. The impact of economies of scope is discussed in Section 5.

2 Equilibrium under intelligence gathering

From the first order conditions of utility maximization, firm i 's inverse demand function in market 1 is given by

$$p_i(q_i, \sum_{j \neq i} q_j) = \alpha_i - q_i - \delta \sum_{j \neq i} q_j, \forall i \in N.$$

Similarly, firm i 's inverse demand function in market 2 is given by:

$$P_i(Q_i, \sum_{j \neq i} Q_j) = \beta_i - Q_i - \delta \sum_{j \neq i} Q_j, \forall i \in N.$$

This allows us to write firm i 's gross of linking costs profit function as:

$$\begin{aligned} \Pi_i(q_i, \sum_{j \neq i} q_j, Q_i, \sum_{j \neq i} Q_j) &= \left(\gamma + \gamma_0 n_i(\mathbf{g}) - \gamma_1 t_i(\mathbf{g}) - q_i - \delta \sum_{j \in N} q_j \right) q_i \\ &\quad + \left(\beta_i Q_i - \delta \sum_{j \in N} Q_j \right) Q_i - \frac{1}{2} (q_i + Q_i)^2. \end{aligned}$$

From the first order conditions of profit maximization the equilibrium quantities produced by each firm $i \in N$ in the two markets, when δ is arbitrarily close to 1, can be written as:

$$\begin{aligned} q_i^* &= \frac{1}{3(n^2+4n+3)} ((2n^2+6n+1)(\gamma_0 n_i(\mathbf{g}) - \gamma_1 t_i(\mathbf{g})) - (2n+5)(\gamma_0 n_{-i}(\mathbf{g}) - \gamma_1 t_{-i}(\mathbf{g}))) \\ &\quad - (n^2+3n-1)\beta_i + (n+4) \sum_{j \neq i} \beta_j + 3\gamma_0(n+2), \end{aligned} \tag{2}$$

$$\begin{aligned} Q_i^* &= \frac{1}{3(n^2+4n+3)} ((n^2+3n-1)(-\gamma_0 n_i(\mathbf{g}) + \gamma_1 t_i(\mathbf{g})) + (n+4)(\gamma_0 n_{-i}(\mathbf{g}) - \gamma_1 t_{-i}(\mathbf{g}))) \\ &\quad + (2n^2+6n+1)\beta_i - t(2n+5) \sum_{j \neq i} \beta_j - 3\gamma_0). \end{aligned}$$

We assume that the parameters γ , γ_0 , γ_1 , β_i , $i = 1, \dots, n$ take values which ensure that the equilibrium quantities are positive. We denote by Π_i^* the stage 2 equilibrium gross profit function of a firm i . This function depends on the number of links formed and received by firm i , the total number of links formed by the other firms as well as the total number of links received by

them in stage 1:

$$\Pi_i^* = \Pi_i^*(n_i(\mathbf{g}), t_i(\mathbf{g}), n_{-i}(\mathbf{g}), t_{-i}(\mathbf{g}))^9 \quad (3)$$

The network \mathbf{g} is an *equilibrium intelligence gathering network* if, for all $i \in N$, we have:

$$\Pi_i^*(n_i(\mathbf{g}), t_i(\mathbf{g}), n_{-i}(\mathbf{g}), t_{-i}(\mathbf{g})) - f n_i(\mathbf{g}) \geq \Pi_i^*(n_i(\mathbf{g}'), t_i(\mathbf{g}'), n_{-i}(\mathbf{g}'), t_{-i}(\mathbf{g}')) - f n_i(\mathbf{g}'), \text{ for all } \mathbf{g}' \in \mathcal{G},$$

with $n_{-i}(\mathbf{g}') = n_{-i}(\mathbf{g})$, $t_i(\mathbf{g}') = t_i(\mathbf{g})$, and $t_{-i}(\mathbf{g}') - t_{-i}(\mathbf{g}) = n_i(\mathbf{g}') - n_i(\mathbf{g})$.

We now provide a complete characterization of the architectures of equilibrium networks. We start by noting a convexity property of the firm's profits with respect to the number of links it establishes, then we state a proposition that uses this property.

Lemma 1 *In an equilibrium network \mathbf{g} , firms establish either 0 links or $n - 1$ links.*

Proof To prove the lemma, let $\Pi_i^*(n_i(\mathbf{g}), t_{-i}(\mathbf{g})) = \Pi_i^*(n_i(\mathbf{g}), \hat{t}_i(\mathbf{g}), \hat{n}_{-i}(\mathbf{g}), t_{-i}(\mathbf{g}))$ where $\hat{t}_i(\mathbf{g})$ and $\hat{n}_{-i}(\mathbf{g})$ are some fixed vectors. Moreover, let $\Delta \Pi_i^*(n_i(\mathbf{g}), t_{-i}(\mathbf{g})) = \Pi_i^*(n_i(\mathbf{g}) + 1, t_{-i}(\mathbf{g}) + 1)$ be the marginal gross profit of firm i from an additional link. By equation 3, we have

$$\Delta \Pi_i^*(n_i(\mathbf{g}) + 1, t_{-i}(\mathbf{g}) + 1) - \Delta \Pi_i^*(n_i(\mathbf{g}), t_{-i}(\mathbf{g})) = 2(\Lambda \gamma_0^2 + \Lambda \gamma_1^2 - \Phi \gamma_0 \gamma_1) > 0. \quad (4)$$

This means that the gross profit function of a firm satisfies a convexity in own links property, i.e. the marginal (gross) returns to a firm from an additional link it forms are increasing in the number of own links already formed. The results follows. \square

Proposition 1 *Let the payoff function satisfy (3). Then there exist numbers F_1, F_2 , where $F_1 > F_2$, with the following properties:*

1. *For $f > F_1$, the empty network is the unique equilibrium intelligence gathering network.*
2. *For $f < F_2$, the complete network is the unique equilibrium intelligence gathering network;*
3. *For $F_1 \leq f \leq F_2$, an equilibrium intelligence gathering network is a k -all-or-nothing-network.*

⁹This function is given in Appendix A.

Proof See Appendix. □

Few remarks are in order here.

Remark 1. *In equilibrium intelligence activities can lead to asymmetric intelligence gathering networks among ex ante symmetric firms.* Note that for a range of parameters, asymmetric networks where some firms have formed $n - 1$ links and other firms have formed no links at all, are equilibrium networks. In fact, Proposition 1 is true even if $\beta_i = \beta$ for all $i \in N$, that is if firms are *ex ante* identical. Hence this result illustrates how intelligence activities can generate substantial asymmetries among firms with regard to the quality of their products and profits.

Remark 2. *Higher quality product in market 2 results in trade-off with intelligence gathering.* The intuition for this result is as follows. When firm i establishes an additional link in market 1, it has an incentive to increase the quantity produced on the first market (“*output effect*”) and due to diseconomies of scope across markets, decrease the quantity of its product sold on the second market (“*cost effect*”). A higher β_i implies a greater loss of revenue resulting from the decrease in Q_i .

Remark 3. *In equilibrium, firms selling the higher quality goods in market 2 may be the ones that engage in intelligence gathering.* Even if ceteris paribus the better quality sold by a firm on market 2 lowers the incentive for this firm to establish links (Remark 2), it is not necessarily the firms with the lowest quality products on market 2 that will do so. This counterintuitive result can be explained as follows. Recall that when the number of firms which have formed $n - 1$ links increases, the marginal payoff of a firm from intelligence gathering decreases. In some situations, where this (latter) negative effect outweighs the positive effect resulting from differences in product quality on market 2, we can observe equilibrium networks where only the firms with higher quality products on market 2 have formed links. The following example illustrates this situation.

Example 1 Assume $n = 10$, $\gamma = 7$, $\gamma_0 = 0.5$, $\gamma_1 = 0.1$, $\beta_i = 6$, for five firms, $\beta_i = 6.1$ for five other firms and $f = 0.45$. We can check that the network where firms having the higher quality

product in market 2 have formed $n - 1$ links on market 1 and the firms having the lower quality product on market 2 have formed no links on market 1 is an equilibrium network.

Remark 4. *Firms may have an incentive to be spied upon.* It is interesting to note that in some situations firms do not have an incentive to protect themselves against intelligence gathering by competitors, as the following example illustrates.

Example 2 Assume $n = 3$, $\gamma = 7$, $\gamma_0 = 0.5$, $\gamma_1 = 0.1$, $\beta_i = 22$, for all $i = 1, 2, 3$, and $f = 0$. Consider a network \mathbf{g} where firms 1 and 2, have formed 2 links each and firm 3 has formed links only with firm 1. We can check that if 3 forms a link with 2, the latter's profits increase.

The intuition for this result stems from the interplay between the “output effect” and the “cost effect”. The example shows that firms have an incentive to be spied upon when the “cost effect” (which increases profits) dominates the “output effect” (which decreases profits). We now establish that, under some conditions, the complete network is not an equilibrium intelligence gathering network when costs of spying are zero.

Corollary 1 *Suppose the payoff function satisfies (3) and the cost of forming links is zero. Then, there exist parameters, γ , γ_0 , $(\beta_i)_{i \in N}$, such that the empty network, and the k -all-or-nothing-networks are equilibrium intelligence gathering networks.*

Proof The proof is straightforward and is omitted. □

This result suggests that even if there are no costs of spying, due to the two effects mentioned above there are instances when firms have no incentive to gather information about other firms, i.e., *the set of equilibrium intelligence gathering networks does not include the complete network*. Note that this result differs from the rest of networks literature where zero link costs always lead to the complete network in equilibrium. This is also true when intelligence gathering occurs in the absence of spillovers across markets as in BB (Proposition 1, pg. 598).

3 Welfare under intelligence gathering

For any network \mathbf{g} , social welfare, $W(\mathbf{g})$, is defined as the sum of consumer surplus and aggregate profits of the n firms.

We define a network \mathbf{g} as *efficient* if $W(\mathbf{g}) \geq W(\mathbf{g}')$ for all $\mathbf{g}' \in \mathcal{G}$.

3.1 Consumer Welfare

We show that consumers surplus is maximized either for the complete network or for the empty network. We begin by showing that consumers' welfare does not depend on the number of intelligence gathering links established by specific firms, but on the total number of links that occur in the industry.

Lemma 2 *Suppose that the utility function satisfies (1) and the quantities produced satisfy (2). The surplus of consumers depends on the total number of links and not on the distribution of links among firms.*

Proof The surplus of consumers is given by:

$$S_C(\mathbf{q}^*(\mathbf{g}), \mathbf{Q}_i^*(\mathbf{g})) = \frac{1}{2} (\sum_{i=1}^n q_i^*(\mathbf{g}))^2 + \frac{1}{2} (\sum_{i=1}^n Q_i^*(\mathbf{g}))^2$$

where

$$\sum_{i \in N} q_i^*(\mathbf{g}) = \frac{(n+2)(\gamma_0 \sum_{i=1}^n n_i(\mathbf{g}) - \gamma_1 \sum_{i=1}^n t_i(\mathbf{g})) - \sum_{i=1}^n \beta_i + (n+2)n\gamma}{4n+3+n^2}.$$

and

$$\sum_{i \in N} Q_i^*(\mathbf{g}) = \frac{-\gamma_0 \sum_{i=1}^n n_i(\mathbf{g}) + \gamma_1 \sum_{i=1}^n t_i(\mathbf{g}) + (n+2) \sum_{i=1}^n \beta_i - n\gamma}{4n+3+n^2}.$$

Since $\sum_{i=1}^n t_i(\mathbf{g}) = \sum_{i=1}^n n_i(\mathbf{g})$, we have $\sum_{i \in N} q_i^*(\mathbf{g}) = \frac{(n+2)(\gamma_0 - \gamma_1) \sum_{i=1}^n n_i(\mathbf{g}) - \sum_{i=1}^n \beta_i + (n+2)n\gamma}{4n+3+n^2}$, and $\sum_{i \in N} Q_i^*(\mathbf{g}) = \frac{(-\gamma_0 + \gamma_1) \sum_{i=1}^n n_i(\mathbf{g}) + (n+2) \sum_{i=1}^n \beta_i - n\gamma}{4n+3+n^2}$.

It follows that the aggregate quantities depend only on the total number of links. As a result, the surplus of consumers does not depend on the pattern of intelligence gathering; it depends only on the aggregate level of this activity. \square

Proposition 2 *Suppose that the utility function satisfies (1), and the quantities produced satisfy (2). The efficient intelligence gathering network for consumers is either the empty network or the complete network.*

Proof Let $S_C(T)$ denote the surplus of consumers in a network \mathbf{g} , where the total number of links formed by the firms is $\sum_{i=1}^n n_i(\mathbf{g}) = T$. We have:

$$S_C(T+1) + S_C(T-1) - 2S_C(T) = (\gamma_0 - \gamma_1)^2 \left(\frac{n^2 + 4n + 5}{(n^2 + 4n + 3)^2} \right) > 0,$$

since $\gamma_0 > \gamma_1$.

Observe that the surplus of consumers exhibits increasing returns with respect to the number of links formed by firms. Hence the efficient network for consumers is either the empty network or the complete network, depending on the sign of the expression $S_C((n-1)^2) - S_C(0)$. \square

Note that consumers may be negatively affected by the spying activities of firms even if it leads to an increase in product quality in market 1. This can be explained in the following way. Link formation in market 1 has two opposite effects on consumers welfare. First, firms offer a better quality product in market 1 and as a whole have an incentive to sell more in this market. This behavior is clearly beneficial to consumers. Second, due to diseconomies of scope, as firms sell more in market 1, they have an incentive to sell less in market 2. This leads to higher prices in market 2 and is harmful for consumers. The above proposition establishes that this latter effect may outweigh the gains from the higher quality in market 1.

3.2 Social Welfare

In this section, to simplify the analysis, we assume that the following condition is satisfied:

$$\text{Condition 1: } \gamma_0 > 3(n-1)\gamma_1,$$

meaning that when a firm i forms an intelligence gathering link to a firm j , the negative impact on the quality of firm j 's product is low relative to the positive impact of the link on the quality of firm i 's product.

Lemma 3 *Let the payoff function satisfy (3), and Condition 1. Let \mathbf{g} be an intelligence gathering network which maximises the aggregate profits of firms. Then, there is at most one firm i such that $n_i(\mathbf{g}) \in \{2, \dots, n-2\}$.*

Proof See appendix. □

Proposition 3 *Let the payoff function satisfy (3), and Condition 1. In an efficient intelligence gathering network \mathbf{g} , there is at most one firm i such that $n_i(\mathbf{g}) \in \{2, \dots, n-2\}$.*

Proof Let the payoff function satisfy (3), and Condition 1. For a contradiction suppose that in an efficient network \mathbf{g} , there are two firms i and j such that $n_i(\mathbf{g}) \in \{2, \dots, n-2\}$ and $n_j(\mathbf{g}) \in \{2, \dots, n-2\}$. By Lemma 3, we know that \mathbf{g} does not maximise the aggregate profits of firms. More precisely, it is possible to increase aggregate profits by a substitution of a link formed by one of the firms i and j with a link formed by the other firm. Let \mathbf{g}' be the networks that results from such a substitution of links. Since \mathbf{g}' contains the same total number of links as \mathbf{g} , we know by Lemma 2, that the surplus of consumers is equal in \mathbf{g} and \mathbf{g}' . As a result the total welfare is higher in \mathbf{g}' than in \mathbf{g} . A contradiction. □

It follows from proposition 3 that only four types of architectures can be efficient: the empty network, the complete network, the k -all-or-nothing networks, and an architecture in which one firm has formed links only with a subset of firms, while each other firm has formed either no links or links with the $n-1$ other firms. Recall that the last architecture cannot be an equilibrium network. In fact, efficient networks and equilibrium networks do not always coincide.

Remark 5. *Conflict between equilibrium and efficient intelligence gathering networks, and policy implication.* Below is a simple example where a conflict between efficiency and equilibrium arises.

Example 3 Assume $n = 3$, $\gamma = 20$, $\gamma_0 = 2$, $\gamma_1 = 0.1$, $\beta_i = 22$, for all $i = 1, 2, 3$, and $f = 6$.

It can be checked that the complete network is an equilibrium intelligence gathering network, but not an efficient intelligence gathering network. For instance, the network where 2 firms

have established 2 links each and 1 firm has formed no links is more efficient than the complete network.

Thus, equilibrium intelligence gathering networks can be over-connected with respect to social welfare leading to over-investment in spying activities in equilibrium. This provides a strong argument for policy intervention with regard to business related intelligence gathering, as the US Industrial Espionage Act of 1996.

Remark 6. *Links setting can have a detrimental effect on welfare even when the cost of links are null, and links have no detrimental effects on the quality of the firms which receive the links.* We may think that when the cost of links are null, and links have no detrimental effects on the quality of the firms which receive the links, links setting is always welfare enhancing. The example below illustrates the fact that it is not always the case. The negative effect of links on welfare is driven by the spillovers across markets.

Example 4 Assume $n = 3$, $\gamma = 7$, $\gamma_0 = 0.5$, $\gamma_1 = 0$, $\beta_i = 73$, for all $i = 1, 2, 3$, and $f = 0$.

It can be checked that the complete network is not an efficient network. For instance, the network where 2 firms have established 2 links each and one firm has no links is more efficient than the complete network.

4 Intelligence Gathering in Both Markets

We now extend our basic model by allowing firms to engage in intelligence gathering activities in both markets. While the basic insights remain the same, we show that the possible range of equilibrium networks increases dramatically since two markets allow for a richer set of outcomes.¹⁰

In the following, we denote by $n_{i\ell}(g)$ the number of links formed by firm i in market ℓ , where $\ell = 1, 2$, and by $t_{i\ell}(g)$ the number of links formed by other firms $j \neq i$ with firm i in market ℓ ,

¹⁰Due to space constraints here we briefly present the results. More details can be found in the supplementary materials to this paper (<https://sites.google.com/site/subhadipchakrabarti/bbcs-paper.1>).

where $\ell = 1, 2$. We assume that the qualities of products sold by firm i in market 1, α_i , and on market 2, β_i , depend on the number of links established in each market in the following way:

$$\begin{aligned}\alpha_i &= \alpha + \alpha_0 n_{i1}(g) - \alpha_1 t_{i1}(g); \\ \beta_i &= \beta + \beta_0 n_{i2}(g) - \beta_1 t_{i2}(g),\end{aligned}$$

with $\alpha, \alpha_0, \alpha_1, \beta, \beta_0, \beta_1$ as positive parameters.

In addition to the parameters introduced above, we introduce the following positive parameters:

$$\begin{aligned}F &= d(110n^2 + 84n^3 + 14n^4 - 16 - 48n); \quad \tau = d(8 + 182n + 124n^2 + 22n^3); \\ \chi &= d(142n + 92n^2 + 14n^3 - 8); \quad \xi = d(158 + 100n + 14n^2),\end{aligned}$$

where $d = (18(n^2 + 4n + 3)^2)^{-1}$. The gross marginal benefit of forming a link in market 1 and in market 2 are given respectively by

$$\begin{aligned}\delta_{i1}(g) &= -\beta_i (\alpha_1 \chi + \alpha_0 F) + \sum_{j \neq i} \beta_j (\alpha_0 \chi + \alpha_1 \xi) \\ &\quad + \alpha_i (2\Lambda \alpha_0 + \tau \alpha_1) - \sum_{j \neq i} \alpha_j (\alpha_0 \tau + 2\Delta \alpha_1) \\ &\quad + \alpha_0^2 \Lambda + \alpha_0 \alpha_1 \tau + \alpha_1^2 \Delta,\end{aligned} \tag{5}$$

$$\begin{aligned}\delta_{i2}(g) &= -\alpha_i (\beta_1 \chi + \beta_0 F) + \sum_{j \neq i} \alpha_j (\beta_0 \chi + \beta_1 \xi) \\ &\quad + \beta_i (2\Lambda \beta_0 + \tau \beta_1) - \sum_{j \neq i} \beta_j (\beta_0 \tau + 2\Delta \beta_1) \\ &\quad + \beta_0^2 \Lambda + \beta_0 \beta_1 \tau + \beta_1^2 \Delta,\end{aligned} \tag{6}$$

where $\Lambda = d(11n^4 + 66n^3 + 107n^2 + 24n + 8)$, and $\Delta = d(11n^2 + 58n + 83)$.

Next, we prove the following lemma.

Lemma 4 *In an equilibrium network, firms will either form zero links or $n - 1$ links in market 1 or market 2.*

Proof We prove this for market 1. The proof for market 2 is analogous. Consider $\delta_{i1}(g)$. From (??), it is clear that this is strictly increasing in α_i and strictly decreasing in $\sum_{j \neq i} \alpha_j$.

Now, with each additional link, α_i increases strictly and $\sum_{j \neq i} \alpha_j$ decreases strictly. So, $\delta_{i1}(g)$ increases strictly. So, if it is worthwhile to form k links, it is also worthwhile to form $k + 1$ links. The result follows. \square

Therefore all equilibrium networks will be complete, empty or such that in each market, each firm has formed either 0 or $n - 1$ links.

In fact, we can say a little bit more about the equilibrium networks that will form. Consider a scenario where in the first market k_1 firms have formed $n - 1$ links and $n - k_1$ firms have formed zero links. Also, in the second market, k_2 firms have formed n links and $n - k_2$ firms have formed zero links. Consider a firm i that has formed zero links in market ℓ , now choosing to form $n - 1$ links. Let the change in profit be denoted by $\Delta \Pi_{i,\ell}^{*k_1,k_2}$. Then, we can show the following.

Lemma 5 $\Delta \Pi_{i,\ell}^{*k_1,k_2}$ is strictly decreasing in k_ℓ .

Proof See appendix. \square

Then we have the following proposition.

Proposition 4 There exists numbers Γ_1, Γ_2 such that:

1. If $f > \Gamma_1$, then the empty network is the unique equilibrium network.
2. If $f < \Gamma_2$, then the complete network is the unique equilibrium network.
3. If $\Gamma_1 \leq f \leq \Gamma_2$, then the equilibrium network is such that in each market, each firm has formed either 0 or $n - 1$ links.

Proof 1. Consider the empty network. Then, a firm i can (from Lemma 4) form $n - 1$ links in market 1, $n - 1$ links in market 2, or both. Let the change in profits be Δ_{i1}, Δ_{i2} and Δ_{i3}

respectively. It can be shown that $\Delta_{i3} - \Delta_{i1} - \Delta_{i2} < 0$. Hence, if $\Delta_{i1} < 0$ and $\Delta_{i2} < 0$, then $\Delta_{i3} < 0$ as well. Now, consider the change in gross profits if a firm forms $n - 1$ links in market ℓ ($\ell = 1, 2$). Call this Υ_ℓ . Let $\Lambda_\ell = \Upsilon_\ell/n - 1$. In that case,

$$\begin{aligned}\Delta_{i\ell} &= \Upsilon_\ell - (n - 1)f \\ &= (n - 1)(\Lambda_\ell - f).\end{aligned}$$

Hence, if $\Gamma_1 = \max(\Lambda_1, \Lambda_2)$, the result follows. Uniqueness follows from Lemma 5 in a manner analogous to the proof of Proposition 1.

2. Consider the complete network. Then, a firm i can (from Lemma 4) delete $n - 1$ links in market 1, $n - 1$ links in market 2, or both. Let the change in profits be Δ_{i1}, Δ_{i2} and Δ_{i3} respectively. Now, consider the change in gross profits if a firm deletes $n - 1$ links in market ℓ ($\ell = 1, 2$). Call this $-\gamma_\ell$. Let $\delta_\ell = \gamma_\ell/(n - 1)$. In that case,

$$\begin{aligned}\Delta_{i\ell} &= -\gamma_\ell + (n - 1)f \\ &= (n - 1)(-\delta_\ell + f).\end{aligned}$$

Also, consider the change in gross profits (or profits not accounting for change in link formation costs) if a firm deletes $n - 1$ links in both markets. Call this $-\gamma_3$. Let $\delta_3 = \gamma_3/(2n - 2)$.

$$\begin{aligned}\Delta_{i3} &= -\gamma_3 + 2(n - 1)f \\ &= 2(n - 1)(-\delta_3 + f).\end{aligned}$$

Hence, if $\Gamma_2 = \min(\delta_1, \delta_2, \delta_3)$, the result follows. Uniqueness follows from Lemma 5 in a manner analogous to the proof of Proposition 1.

3. The third part of the proposition follows in a straightforward manner from the first two parts of Proposition 4 and Lemma 4.

□

Next, we focus on the aspect that is most important from a policy perspective, and illustrates the role of externalities: the conflict between efficiency and equilibrium. This conflict continues

to be present when we introduce the possibility of firms spying in both markets. In particular, the following example illustrates how firms can over-invest in spying links with regard to social welfare.

Example 5 This is similar to Example 3 and exploits continuity by making β_0 sufficiently small. Assume $n = 3$, $\alpha = 20$, $\alpha_0 = 2$, $\beta = 22$, $\beta_0 = 0.000001$, $\alpha_1 = \beta_1 = 0$, and $f = 6$. The network \mathbf{g} in which all firms form two links in market 1 and no links in market 2 is an equilibrium but not an efficient network. Indeed the network in which two firms form two links each and the other firms form no links in market 1, and no firm forms any links in market 2 is more efficient than the network \mathbf{g} .

Remark 7. *Seemingly collusive behavior can arise in equilibrium.* It is worth noting that for certain parameters ranges firms may choose to spy on their rivals only on one market. This leads to a situation where every firm improves its quality and behaves aggressively on one market, only allowing its competitors to do the same on the other market. Note that this seemingly collusive behavior arises in equilibrium.

Example 6 Assume $n = 4$, $\alpha = 1500$, $\alpha_0 = 2$, $\alpha_1 = 0$, $\beta = 1500$, $\beta_0 = 2$, $\beta_1 = 0$, and $486 \leq f \leq 495.56$. The network \mathbf{g} in which firms 1 and 2 form three links each in market 1 and no links in market 2 and firms 3 and 4 form three links each in market 2 and no links in market 1 is an equilibrium network.

5 Economies of Scope versus Diseconomies of Scope

We now explain what happens when there exist positives spillovers across markets, i.e., the cost function exhibits economies of scope. We show that in this case there is no tension across markets anymore and firms have always an incentive to form links and gather information, provided costs of links are low enough. For simplicity we assume that firms spy in only one market, though as before the insights can be generalized to allow for spying on multiple markets. In equilibrium

we have:

$$\frac{d\pi_i^*}{d\alpha_i} = \sum_{j \neq i} \left(\frac{\partial \pi_i^*}{\partial q_j^*} \right) \left(\frac{dq_j^*}{d\alpha_i} \right) + \sum_{j \neq i} \left(\frac{\partial \pi_i^*}{\partial Q_j^*} \right) \left(\frac{dQ_j^*}{d\alpha_i} \right) + \frac{\partial \pi_i^*}{\partial \alpha_i},$$

where π_i^* is equilibrium profit, and q_j^*, Q_j^* , are equilibrium quantities.

Unlike the case with diseconomies of scope across markets, in this case $\frac{\partial Q_j^*}{\partial \alpha_i}$ is positive. Since the three terms in the above expression have a positive sign, firms always have an incentive to spy in order to increase the quality of their products when intelligence gathering is costless. The intuition behind this result is as follows: when firm i forms a link and increases the quality of its product on a market, then it adopts a more aggressive strategy not only on this market, but on the other market too due to economies of scope. Since competitors regard their product as a strategic substitute for the products of firm i on each market, they sell less on both markets and this behavior is beneficial to firm i . Hence the complete network is the only possible equilibrium intelligence gathering network with zero costs of spying. When costs of spying are positive, then the equilibrium architecture is dependent only on the value of f and not on the spillovers across markets, making economies of scope rather uninteresting in the multimarket context.

Conclusion

In this paper we study the incentives of firms to gather information about each other in order to increase the quality of their products. This is done in a multimarket setting where competitors regard their products as strategic substitutes. A significant finding is that under diseconomies of scope firms may have no incentive to gather information about each other even if intelligence activities are costless. Moreover, in some situations firms might even prefer other firms to gather information on them. We find that although intelligence activities lead to increased quality products, they may lead to a reduction of social welfare as well as consumers welfare. In other words, in some cases, due to the presence of significant externalities, equilibrium level of intelligence gathering can exceed the socially optimal level, making a strong case for regulatory intervention.

Our paper is the first formal analysis of competitive intelligence type activities which are

becoming increasingly important in modern economies. We briefly discuss some issues that could be explored in future work. First we take up the issue of intelligence gathering activities. In our model intelligence gathering is always successful. However in future work intelligence gathering needs only be successful with a certain probability. The second issue is the impact of intelligence gathering activities. An important question would be to examine the consequences of such activities for future product development and its impact on social welfare. It might also be interesting to examine the impact of such activities when firms play a price game. Third, from a network perspective we need to examine multimarket competition with inter-related costs where firms make collaborative R &D decisions. In this case it would be necessary to modify the equilibrium concept to allow for consent. This would enable us to consider other stability notions like the notion of pairwise stability proposed by Jackson and Wolinsky (1996, [8]).

Appendix A. Intelligence gathering in one market: Values of profit parameters

In the following, let $d = (18(n^2 + 4n + 3)^2)^{-1}$. The equilibrium gross profits in stage 2 is

$$\begin{aligned}
\Pi_i^* = & \Lambda(\gamma_0^2 n_i(\mathbf{g}) - \gamma_1^2 t_i(\mathbf{g}))^2 + \Delta(\gamma_0^2 n_{-i}(\mathbf{g})^2 + -\gamma_1^2 t_{-i}(\mathbf{g}))^2 \\
& + \Phi(\gamma_0^2 n_i(\mathbf{g}) n_{-i}(\mathbf{g}) + \gamma_1^2 t_i(\mathbf{g}) t_{-i}(\mathbf{g}) - \gamma_0 \gamma_1 n_i(\mathbf{g}) t_{-i}(\mathbf{g}) - \gamma_0 \gamma_1 n_{-i}(\mathbf{g}) t_i(\mathbf{g})) \\
& + \Omega_i(\gamma_0 n_i(\mathbf{g}) - \gamma_1 t_i(\mathbf{g})) + \Gamma(\gamma_0 n_{-i}(\mathbf{g}) - \gamma_1 t_{-i}(\mathbf{g})) + \Psi_i,
\end{aligned} \tag{7}$$

with

$$\begin{aligned}
\Lambda &= d(11n^4 + 66n^3 + 107n^2 + 24n + 8), \\
\Phi &= -2d(11n^3 + 62n^2 + 91n + 4), \\
\Omega_i &= 2d(3\gamma(5n^3 + 26n^2 + 37n + 4) - (7n^4 + 42n^3 + 55n^2 - 24n - 8)\beta_i \\
&\quad + (n + 4)(7n^2 + 18n - 1)\sum_{j \neq i} \beta_j), \\
\Psi_i &= d(\gamma((9\gamma(3n^2 + 10n + 11) - 6(n^3 + 10n^2 + 17n - 4)\beta_i + 6(n^2 + 14n + 25)\sum_{j \neq i} \beta_j) \\
&\quad - 2(11n^3 + 62n^2 + 91n + 4)\beta_i \sum_{j \neq i} \beta_j + (11n^4 + 66n^3 + 107n^2 + 24n + 8)\beta_i^2 \\
&\quad + (11n^2 + 58n + 83)(\sum_{j \neq i} \beta_j)^2), \\
\Gamma_i &= -2d(\gamma(15n^2 + 66n + 87) - (n + 4)(7n^2 + 18n - 1)\beta_i + (7n^2 + 50n + 79)\sum_{j \neq i} \beta_j), \\
\Delta &= d(11n^2 + 58n + 83).
\end{aligned}$$

We have $\Lambda > 0$, $\Delta > 0$, $\Phi < 0$, $\Gamma_i \in \mathbb{R}$, $\Psi_i \in \mathbb{R}$. Moreover, note that $\Omega_i = \Omega_i(\beta_i, \sum_{j \neq i} \beta_j)$ is decreasing in its first argument and increasing in its second argument.

Appendix B. Proofs.

Proof of Proposition 1

(i) To simplify presentation, we write $\Pi_i(\mathbf{g})$ as $\Pi_i(n_i(\mathbf{g}), t_i(\mathbf{g}), \sum_{j \neq i \in N} n_j(\mathbf{g}), \sum_{j \neq i \in N} t_j(\mathbf{g}))$. We know by Lemma 1 that each firm $i \in N$ forms either 0 links or $n - 1$ links in an equilibrium intelligence gathering network. (ii) let $\Delta \Pi_i^{*k} = \Pi_i^*(n - 1, k, k(n - 1), k(n - 2) + n - 1) - \Pi_i^*(0, k, k(n - 1), k(n - 2))$ be the marginal gross profit to a firm i from forming $n - 1$ links in a network where k firms have formed $n - 1$ links. We have:

$$\Delta \Pi_i^{*k} - \Delta \Pi_i^{*k-1} = -(n - 1)((\gamma_0 - \gamma_1)(n - 1) + \gamma_1)(2\gamma_1\Delta - \Phi\gamma_0) - \Phi\gamma_1^2 + 2\Lambda\gamma_0\gamma_1 < 0,$$

since $\gamma_0 > 0$, $\gamma_1 > 0$, $\gamma_0 > \gamma_1$, $\Delta > 0$, $\Lambda > 0$, and $\Phi < 0$.

This result means that the marginal benefit from forming $n - 1$ links is decreasing with the number of firms which have already formed $n - 1$ links. It follows that if firms has no incentive to form links in the empty network, then in any non empty network, firms have an incentive to delete links. Moreover if firms have no incentive to delete links in the complete network, then in any network firms have an incentive to form links. The two first parts of the proposition follows

this statement.

(1) The empty network is the unique equilibrium network if and only if no firm $i \in N$ has any incentive to form $n - 1$ links in this network. We have

$$\Pi_i^*(n - 1, 0, 0, n - 1) - \Pi_i^*(0, 0, 0, 0) \leq f(n - 1),$$

that is

$$\frac{1}{18(n^2 + 4n + 3)^2}((n - 1)(\gamma_0^2\Lambda + \gamma_1^2\Delta - \gamma_0\gamma_1\Phi) + \gamma_0\Omega_i - \gamma_1\Gamma_i) \leq f$$

Let $\underline{\Gamma} \in \{\Gamma_1, \dots, \Gamma_n\}$ and $\underline{\Omega} \in \{\Omega_1, \dots, \Omega_n\}$ be such that $(\underline{\Gamma}, \underline{\Omega}) = \arg \max_{(\Omega_i, \Gamma_i)} \gamma_0\Omega_i - \gamma_1\Gamma_i$, and $F_1 = (n - 1)(\gamma_0^2\Lambda + \gamma_1^2\Delta - \gamma_0\gamma_1\Phi) + (\gamma_0\underline{\Gamma} - \gamma_1\underline{\Omega})$. The result follows.

(2) The complete network is the unique equilibrium network if and only if no firm $i \in N$ has any incentive to delete $n - 1$ links in this network. We have

$$\Pi_i^*(n - 1, n - 1, (n - 1)^2, (n - 1)^2) - \Pi_i^*(0, n - 1, (n - 1)^2, (n - 1)(n - 2)) \geq f(n - 1),$$

that is

$$\begin{aligned} & \frac{1}{18(n^2 + 4n + 3)^2}((n - 1)(\gamma_0(\gamma_0 - 2\gamma_1)\Lambda - \gamma_1(2\gamma_0(n - 1) - \gamma_1(2n - 3))\Delta \\ & + (\gamma_0^2(n - 1) - \gamma_1(n\gamma_0 - \gamma_0 - \gamma_1))\Phi) + \gamma_0\Omega_i - \gamma_1\Gamma_i) \geq f \end{aligned}$$

Let $\bar{\Gamma} \in \{\Gamma_1, \dots, \Gamma_n\}$ and $\bar{\Omega} \in \{\Omega_1, \dots, \Omega_n\}$ be such that $(\bar{\Gamma}, \bar{\Omega}) = \arg \min_{(\Omega_i, \Gamma_i)} \gamma_0\Omega_i - \gamma_1\Gamma_i$, and $F_2 = (n - 1)(\gamma_0(\gamma_0 - 2\gamma_1)\Lambda - \gamma_1(2\gamma_0(n - 1) + \gamma_1(3 - 2n))\Delta + (\gamma_0^2(n - 1) - \gamma_1(n\gamma_0 - \gamma_0 - \gamma_1))\Phi) + \gamma_0\bar{\Omega} - \gamma_1\bar{\Gamma}$. The result follows.

(3) The third part of the proposition follows in a straightforward manner from the first two parts of the proposition and Lemma 1.

□

Proof of Lemma 3 Suppose the payoff function satisfy (3), and condition 1. Let \mathbf{g} be an efficient network for multimarket firms. Assume there are two firms, say i and j in \mathbf{g} , such that $n_i(\mathbf{g}) \in \{1, \dots, n - 2\}$ and $n_j(\mathbf{g}) \in \{1, \dots, n - 2\}$. Without loss of generality, suppose that i and j

are such that if $n_i(\mathbf{g}) \neq n_j(\mathbf{g})$, then $n_i(\mathbf{g}) > n_j(\mathbf{g})$, and if $n_i(\mathbf{g}) = n_j(\mathbf{g})$ and $\max\{g_{i,j}, g_{j,i}\} = 1$, then $g_{i,j} = 1$. We show that it is possible to increase the total profits thanks to the substitution of a link formed by i to a link formed by j . In the following, let $Y = 2(\Lambda + \Delta - \Phi)$. We have $Y > 0$, since $\Lambda > 0$, $\Delta > 0$ and $\Phi < 0$. There are three cases.

Case 1: There exists in \mathbf{g} a firm $k \in N \setminus \{i, j\}$ such that $g_{j,k} = 1$ and $g_{i,k} = 0$. Suppose j deletes its link with k and i adds a link with k in \mathbf{g} , while the other firms do not change their links. Let \mathbf{g}' be the resulting network. We have:

$$\begin{aligned} \sum_{i \in N} \Pi_i^*(\mathbf{g}) - \sum_{i \in N} \Pi_i^*(\mathbf{g}') &= Y\gamma_0(\gamma_0(n_i(\mathbf{g}) - n_j(\mathbf{g}) + 1) - \gamma_1(t_i(\mathbf{g}) - t_j(\mathbf{g}))) \\ &> Y\gamma_0(\gamma_0 - (n-1)\gamma_1) \\ &> Y\gamma_0(\gamma_0 - 3(n-1)\gamma_1) \\ &> 0, \end{aligned} \tag{8}$$

Thus, \mathbf{g} does not maximize the aggregate profits of firms, giving us a contradiction.

Case 2. For all $k \in N \setminus \{i, j\}$, if $g_{j,k} = 1$ then $g_{i,k} = 1$. But there are firms $\ell, m \in N \setminus \{i, j\}$ such that $g_{i,\ell} = g_{j,\ell} = 1$, and $g_{i,m} = g_{j,m} = 0$. Suppose j deletes its link with ℓ and i adds a link with m in \mathbf{g} , while the other firms do not change their links. Let \mathbf{g}' be the resulting network. We have:

$$\begin{aligned} \sum_{i \in N} \Pi_i^*(\mathbf{g}) - \sum_{i \in N} \Pi_i^*(\mathbf{g}') &= (\gamma_0^2(n_i(\mathbf{g}) - n_j(\mathbf{g}) + 1) + \gamma_1^2(t_\ell(\mathbf{g}) - t_m(\mathbf{g}) + 1) \\ &\quad + \gamma_0\gamma_1(n_m(\mathbf{g}) - n_\ell(\mathbf{g}) + t_j(\mathbf{g}) - t_i(\mathbf{g}))) \\ &> \gamma_0^2 - (n-2)\gamma_1^2 - (2n-2)\gamma_0\gamma_1 \\ &> \gamma_0(\gamma_0 - (3n-4)\gamma_1) \\ &> \gamma_0(\gamma_0 - 3(n-1)\gamma_1) \\ &> 0, \end{aligned} \tag{9}$$

Thus, \mathbf{g} does not maximize the aggregate profits of firms, giving us a contradiction.

Case 3. For all $k \in N \setminus \{i, j\}$, if $g_{j,k} = 1$ then $g_{i,k} = 1$. Moreover for all firms $\ell, m \in N \setminus \{i, j\}$, if $g_{i,\ell} = g_{j,\ell} = 1$ then $g_{i,m} = g_{j,m} = 1$. Therefore, either $g_{i,m} = g_{j,m} = 1$ for all firms $\ell, m \in N \setminus \{i, j\}$ and $g_{i,j} = 0$, or $N_j(\mathbf{g}) = \{i\}$.

First, suppose $g_{i,m} = g_{j,\ell} = 1$ for all firms $\ell, m \in N \setminus \{i, j\}$ and $g_{i,j} = 0$. Suppose j deletes its link with $k \in N \setminus \{i, j\}$ and i adds a link with j in \mathbf{g} , while the other firms do not change their links. Let \mathbf{g}' be the resulting network. We have:

$$\begin{aligned}
\sum_{i \in N} \Pi_i^*(\mathbf{g}) - \sum_{i \in N} \Pi_i^*(\mathbf{g}') &= \gamma_0^2 + \gamma_1^2(t_j(\mathbf{g}) - t_k(\mathbf{g}) + 1) \\
&\quad + \gamma_0\gamma_1(n_k(\mathbf{g}) - n_j(\mathbf{g}) + t_j(\mathbf{g}) - t_i(\mathbf{g}) + 1)) \\
&> \gamma_0^2 - (n-2)\gamma_1^2 - (2n-1)\gamma_0\gamma_1 \\
&> \gamma_0(\gamma_0 - 3(n-1)\gamma_1) \\
&> 0,
\end{aligned} \tag{10}$$

Thus, \mathbf{g} does not maximize the aggregate profits of firms, giving us a contradiction.

Suppose now that $N_j(\mathbf{g}) = \{i\}$. Suppose j deletes its link with i and i adds a link with j in \mathbf{g} , while the other firms do not change their links. Let \mathbf{g}' be the resulting network. We have:

$$\begin{aligned}
\sum_{i \in N} \Pi_i^*(\mathbf{g}) - \sum_{i \in N} \Pi_i^*(\mathbf{g}') &= (\gamma_0^2(n_i(\mathbf{g})1) + \gamma_1^2(t_j(\mathbf{g}) - t_i(\mathbf{g}) + 1) \\
&\quad + \gamma_0\gamma_1(n_i(\mathbf{g}) + t_j(\mathbf{g}) - t_i(\mathbf{g}) + 1)) \\
&> \gamma_0^2 - n\gamma_1^2 - 2n\gamma_0\gamma_1 \\
&> \gamma_0(\gamma_0 - (2n-3)\gamma_1) \\
&> \gamma_0(\gamma_0 - 3(n-1)\gamma_1) \\
&> 0
\end{aligned} \tag{11}$$

Thus, \mathbf{g} does not maximize the aggregate profits of firms, giving us a contradiction.

Proof of Lemma 5

We prove the lemma for market 1. The proof for market 2 follows analogously. With a slight abuse of notation we will use the derivative to examine the impact of the number of firms in market 1, k_1 , on the gross marginal benefit of forming $(n-1)$ links in this market, $\Delta\Pi_{i,1}^{*k_1,k_2}$. It can be shown that

$$\frac{\partial \Delta\Pi_{i,1}^{*k_1,k_2}}{\partial k_1} = -\frac{(n-1)(\alpha_0^2\nu_1 + \alpha_0\alpha_1\nu_2 + \alpha_1^2\nu_3)}{9(n^2 + 4n + 3)} < 0,$$

with $\nu_1 = 11n^4 + 51n^3 + 29n^2 - 87n - 4$, $\nu_2 = 37n^3 + 187n^2 + 227n - 67$, and $\nu_3 = 26n^2 + 124n + 170$.

The result follows.

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